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Relativistic Quark-Diquark Model For The Nucleon

Modelo Relativistico Quark-Diquark para el Nucleón

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Resumen

Desarrollamos un modelo constituido de quarks para el nucleón y sus resonancias usando un potencial de oscilador armónico para la interacción. Los efectos debidos a la corrección relativista en la energía cinética son estudiados. Finalmente, el factor de forma eléctrico del modelo es calculado y comparado con los datos experimentales.

Palabras Clave: Efectos relativistas, quark-diquark model.

Abstract

We developed a constituent quark-diquark model for the nucleon and its resonances using a harmonic oscillator potential for the interaction. The effects due tu relativistic kinentic energy correction are studied. Finally, charge forn factor of the model is calculated and compared with experimental data.

Keywords: Relativistic effects, modelo de quark-diquark.

1. Introduction

In phenomenological quarks models framework (CQM), many works have been realized [1] with a non-relativistic approach giving satisfactory results for the static properties of the nucleon and of its excited states. However it has been also proposed a relativistic version of the model in order to improve the theoretical predictions [2], [5], [6].
The main objective of this work is to develop an effective quark-diquark model, also introducing in a perturbative way the kinetic energy relativistic correction. We test how the model mass spectrum is affected by this correction. The diquark is assumed to be a single particle [2], [3], with quantum numbers corresponding to two coupled quarks. This approximation has given good results and also presents the advantage of reducing the three-body problem to a two-body one.
Finally, we will calculate the proton charge form factor by means of the wave function obtained fitting the spectrum. It will be compared with the experimental data. The magnetic form factor will not be studied in this work because it would required a more detailed relativistic model.

2. The Hamiltonian and the wave functions

Taking advantage of some existing nonrelativistic models, we propose the following Hamiltonian with relativistic corrections for the present quark-diquark model

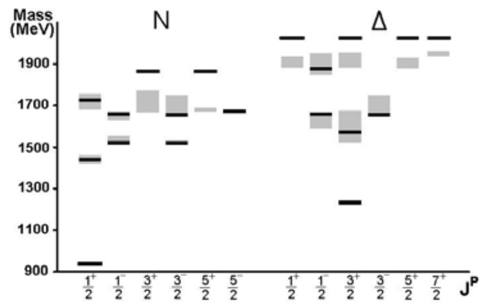

Figura 1. The calculated masses(black lines) and experimental masses (grey boxes)

	N(1440)	N(1535)	N(1680)
ΔE_{cal}(MeV)	697.5	700.7	1120
ΔE_{exp}(MeV)	492-532	582-617	712-812

Cuadro 1
The calculated masses by the quark-diquark model up to 2 GeV

$$H = E_0 + \frac{p^2}{2m} + \frac{1}{2}kr^2 + (B + D\delta_{N,0})\delta_{s_{12},1} + C\delta_{N,0} + 2A(-1)^{l+1}e^{-\lambda^2 r^2}[\vec{s_{12}} \cdot \vec{s_3} + \vec{t_{12}} \cdot \vec{t_3} + 2\vec{s_{12}} \cdot \vec{s_3}\vec{t_{12}} \cdot \vec{t_3}] - \frac{\rho}{r} - \frac{p^4}{8\eta^3} \tag{1}$$

where the first term is the quark-diquark rest energy, the second term is the kinetic energy in terms of the reduced mass m and third one is the harmonic oscilator (analytically solvable) confining potential (HO). The next term is used for a phenomenological description of the $N - \Delta$ splitting. In other models this mass difference is provided by the hyperfine interaction [1], [4]. Then, we have the exchange interaction that has been already used in others works [2], [4]. A Coulomb-like short distance term should be present in the q-q interaction. For this reason we have inserted the ρ/r term. Finally, the p^4 term represents the relativistic correction to the kinetic energy.

Since the HO potential is rotationally invariant, the spatial wave functions can be written as

$$\Psi_{n,l,m}(\vec{r}) = R_{n,l}(r)Y_{l,m}(\theta, \varphi) \tag{2}$$

where the radial wave functions $R_{n,l}$ are obtained analytically in the form

$$R_{n,l}(r) = \left[\frac{2(\frac{n-l}{2})!}{\Gamma(\frac{n+l+3}{2})}\right]^{\frac{1}{2}} \alpha^{-\frac{3}{2}}(\xi)^l e^{-\frac{\xi^2}{2}} L^{l+\frac{1}{2}}_{\frac{n-l}{2}}(\xi^2) \tag{3}$$

α is the dimensional parameter of the wave function, related to the oscillator energy by $\alpha^2 = m\omega$; ξ is the dimensionless variable that is defined as $\xi = \alpha r$. $L^{l+\frac{1}{2}}_{\frac{n-l}{2}}(\xi^2)$ are the well-known Laguerre associate polynomials [8]. The energy eigenvalues corresponding to spatial wave functions are

$$E_{N,l} = (N + \frac{3}{2})\omega \qquad N = 0, 1, 2, 3... \qquad l = N,\ N-2,\ N-4, ...\ (l \geq 0) \tag{4}$$

The spin and isospin wave functions can be obtained by coupling the corresponding quark-diquark quantities.

$$\chi^{s_{12}}_{S,S_3} = \left[\chi^{(1)}_{s_{12}} \otimes \chi^{(2)}_{\frac{1}{2}}\right]_{S,S_3} \qquad G_{S,S_3;T,T_3} = \chi^{s_{12}}_{S,M_s}\phi^{t_{12}}_{T,T_3} \tag{5}$$

where the diquark spin and isospin are $s_{12} = t_{12} = 0, 1$.

3. The spectrum of the model

Using the wave functions written above, we have the following energy spectrum

$$E = E_0 + (N + \frac{3}{2})\omega + (B + D\delta_{N,0})\delta_{s_{12},1} + C\delta_{N,0} + 2A(-1)^{l+1}F_1(\alpha, \lambda)F_2(S, T, s_{12}, t_{12}) - F_3(\beta, \eta) - F_4(\rho, \alpha) \tag{6}$$

The functions F_1, F_2, F_3, F_4 are matrix elements calculated in perturbative way in the HO basis. Their explicit expressions are

$\frac{\beta^{-4}}{16\eta^3}$	N(1440)	N(1535)	N(1680)
74	682.4	695.68	1108
78	622.83	675.68	1066
80	592.84	665.68	1036
80.5	583.34	663.17	1030

Cuadro 2
The relativistic correction fit for three particles that exhibit of good way this effect

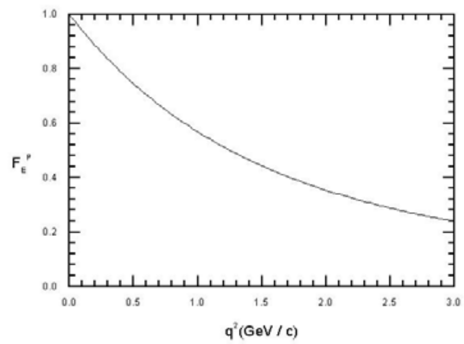

Figura 2. Proton charge form factor

$$F_1 = \int_0^\infty R_{n,l}(r) e^{-\lambda^2 r^2} R_{n,l}(r) r^2 dr \qquad (7)$$

$$F_2 = [\langle \vec{s_{12}} \cdot \vec{s_3} \rangle + \langle \vec{t_{12}} \cdot \vec{t_3} \rangle + 2\langle \vec{s_{12}} \cdot \vec{s_3} \rangle \langle \vec{t_{12}} \cdot \vec{t_3} \rangle] \qquad (8)$$

$$F_3 = \int_0^\infty R_{n,l}(p) \frac{p^4}{8\eta} R_{n,l}(p) p^2 dp \qquad (9)$$

$$F_4 = \int_0^\infty R_{n,l}(r) \frac{\rho}{r} R_{n,l}(r) r^2 dr \qquad (10)$$

The function F_3 is calculated with the wave functions in the momentum representation. The function F_2 can be calculated by means of the Lande equation. [1]. The energy spectrum is characterized by some free parameters. The values that have been chosen for those parameters are $E_0 = 574$ MeV, $\omega = 840$ MeV, $B = 245$ MeV, $D = 207$ MeV, $C = 85$ MeV, $A = 125$ MeV, $\alpha = 1{,}875\ fm^{-1}$, $\lambda = 1{,}64\ fm^{-1}$, $\frac{\beta^{-4}}{16\eta^3} = 80{,}5$ MeV, $\rho = 320$ MeV.fm^{-1}. The result is shown in the next figure The model has given a good description in the lower part of the spectrum, however for high energy states we find some discrepancies with respect to experimental data [7]. This fact can be due to relativistic and field effects that have not been taken into account by the model. In order to search how the relativistic corrections affect the model spectrum, we will analyze the effect of the parameter value $\frac{\beta^{-4}}{16\eta^3}$. Specifically, we are going to study the states N(1440), N(1535), N(1680). We started by using the value $\frac{\beta^{-4}}{16\eta^3}$=73 MeV, the obtained result is shown in table 1.

There are some differences with respect to the experimental interval. Therefore, we changed this parameter until that we obtained the best result. Such fit gives

However, we have stopped the fit of the relativistic correction strenght at the value 80.5 MeV, to keep the calculation in a correct perturbative regime. A different approach, as a variational method, should be used to study large relativistic effects.

4. Proton charge form factor

The form factor is defined as the mean value of the Fourier trasform of the composite system electric charge density [1]. In the nucleon case, the electric form factor for quark-diquark model has the following expression

$$F_E(\vec{q}) = e_1 e^{-(\frac{m_2 q}{2\alpha M})^2} + e_2 e^{-(\frac{m_1 q}{2\alpha M})^2} \qquad (11)$$

we use the parameters obtained in the spectrum fit, $m_1 = 626{,}6$ MeV, $m_2 = 313{,}3$ MeV, M=940 MeV, $\alpha = 370$ MeV. For the proton, the electric factor form is displayed in the figure 2. We find that, as usual, the HO form factor calculation decreases faster than the experimental data. We conclude noting that the results

[1] $\langle \vec{s_{12}} \cdot \vec{s_3} \rangle = \frac{1}{2}[S(S+1) - s_{12}(s_{12}+1) - s_3(s_3+1)]$ The same way for the isospin$\langle \vec{t_{12}} \cdot \vec{t_3} \rangle$

of this work motivate a fully relativistic study of the quark-diquark model for the nucleon.

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