REVISTA COLOMBIANA DE FÍSICA, VOL.37, No.2, 2005

SURFACE WAVE PROPAGATION ALONG A PHOTOREFRACTIVE MEDIA INTERFACE

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(Recibido 11 Ago. 2005; Aceptado 3 Nov. 2005; Publicado 23 Dic. 2005)

RESUMEN

Se presenta una solución exacta de la ecuación diferencial de onda de difusión, para una distribución eléctrica de la función esférica de Bessel $j_0(\mathfrak{a}x)$, como también sus condiciones de frontera, donde este efecto es obtenido por el mecanismo no lineal de difusión en cristales fotorefractivos.

La existencia de la onda fotorefractiva superficial la cual puede ser guiada a lo largo de la interface de los dos cristales fotorefractivos similares pero con diferente signo del coeficiente no lineal de difusión, donde se analiza cuando el radio del haz de entrada $\mathfrak{a}$ y el coeficiente de difusión γ estan relacionados entre si, también se demuestra que las oscilaciones de la distribución de campo eléctrico dada como la función esférica de Bessel son suavizadas cuando el coeficiente no lineal de difusión es incrementado.

Palabras clave: Propagación de ondas, Medio Fotorrefractivo.

ABSTRACT

We present the exact solution to the diffusion wave differential equation for the Spherical Bessel electric field distribution $j_0(\mathfrak{a}x)$, and its boundary conditions, whose effect on diffusion in photorefractive crystals is well characterized by its nonlinearity.

The photorefractive surface wave can be guided along the boundary of two similar photorefractive crystals with different signs of the non linear coefficient of difussion. The relationship between the input beam radius $\mathfrak{a}$ and the diffusion coefficient γ is discussed in this paper as well. Further analysis also revealed that the oscillations of the electric field wave, expressed mathematically by the Spherical Bessel equation, are smoothened as the non linear coefficient of difussion increases.

keywords: Wave propagation, Photorrefractive media.

1. Introduction

Surface waves are a very attractive concept from the theoretical point of view because they make possible the analysis of how they propagate along a photorefractive media interface. Though they are only one kind of the many different kinds of waves that can be localized through the spatial localization of the wave field energy [1].

One of the most important features of surface waves is that when they reach the limit of the non-stationary wave localization they transform themselves into well-know spatial soliton-like's (surface waves) [1, 2, 3, 4].

Several experimental works dealing with surface waves propagation in photorefractive crystals have been successfully performed [2, 5, 6, 7].

The influence of the spatially non-local diffusion component on the nonlinear response gives an additional dielectric constant contribution, which is proportional to the derivative of the light intensity on the transverse coordinate [1]. The physical manifestation of the effect just mentioned is the well-known beam self-bending. Specific features of the self-bending of photorefractive surface waves have been studied in [2, 3, 4] as well.

The diffusion photorefractive nonlinearity effect in which the generation of a photorefractive surface wave is considered , speeds up the photorefractive response. The response speed of the photorefractive effect is limited by the photorefractive non-linearity. It has been predicted that for a photorefractive crystal, such as $BaTiO_3$, the diffusion mechanism can assure efficient concentration of light power in crystal layers of $\sim 10\mu m$ thickness [1].

The optical confinement in waveguide structures allows high intensity to be maintained in the waveguide. An increase in the intensity-length product leads to an appreciable decrease in the effective photorefractive response time for a given input power. The expected decrease in the photorefractive response time as a result of the optical confinement in the waveguide has been observed, along with an unexpected change in the coupling direction [8].

The primary goal of our paper is to obtain an analytical approach in order to determine when the beam propagates along the interface of two photorefractive media. If it is possible to confine the light, then the response would be the formation of surface waves, in which the interface acts as the wave guide. However the interface can also stabilize the surface wave because of the wave deflection in each medium. This paper analyses the case in which light waves undergo self-trapping between photorefractive media interface as well.

For the incident beam, the Maxwell equations are applied between an air-solid media interface of propagation. The solid media interface is usually constituted by two photorefractive crystals (PRC'^s), with opposite sign of the non-linear diffusion. For the beam propagation along the interface of the $PRC' - PRC$ media, the electric field continuity equations and its field derivative function are applied. Here is mathematically demonstrated that the Spherical Bessel function represents the amplitude of the electric field distribution, and that the oscillations are smoothened by the effect of the non linear diffusion of the media when they propagate along the media interface, when the diffusion nonlinearity of the photorefractive media is increased [1, 2, 3, 4].

Christodoulides et. al. showed that the diffusion PRC nonlinearity smoothens the wave oscillations, as is shown in the figure 1 of his paper [9].

This paper is concerned with the control of the light confinement in the media, with low energy loss by spreading, i.e. how to manipulate the physical parameters related to this control (diffusion coefficient), that is possible due to the energy manipulation of the incident beam and the confinement of the energy in the media. The energy manipulation leads to a solution which, although approximate, is in an analytic form and enables direct interpretation of the obtained results.

The content of the paper is organized as follows. In section 2 the theoretical model is formulated, it is developed in order to obtain the differential equation, which describes the electric amplitude behavior in each medium and the continuity conditions that satisfy the Maxwell equations in the interface media. Section 3 describes the required conditions to obtain the surface wave formation to describe the steady solutions. Finally, Conclusions are drawn in section 4.

2. Theoretical model

We model a bidimensional case light beam propagation using the wave equation:

$$\frac{\partial^2 E_1(X,Z,t)}{\partial X^2} + \frac{\partial^2 E_1(X,Z,t)}{\partial Z^2} - \mu\varepsilon_0\varepsilon(X,Z)\frac{\partial^2 E_1(X,Z,t)}{\partial t^2} = 0 \qquad (1)$$

where $E_1(X,Z,t) = E(X,Z)\exp(-i\omega t + i\beta Z)$ is a monochromatic wave with frequency ω, β is the wave propagation constant, and $\varepsilon(X,Z)$ is a real dielectric constant with slow spatial variations. The author of the present paper prefers to use the sign convention of $\exp(-i\omega t + i\beta Z)$ instead of $\exp(i\omega t - i\beta Z)$, which is sometimes used by others; see e.g., [10, 11].

We assume spatial changes given by $\varepsilon(X,Z) = \varepsilon_{\rm a} + \delta\varepsilon(X,Z)$ [1], where $\varepsilon_{\rm a}$ is the dielectric constant of a linear medium.

The photoinduced variation of the dielectric constant is given by

$$\delta\varepsilon(X,Z) = 2n^4 r\frac{K_B T}{e}\frac{\partial \ln|E(X,Z)|}{\partial X} = n^4 r E_{sc}(X,Z) = \varepsilon_{\rm a}^2 r E_{sc}(X,Z) \qquad (2)$$

where E_{SC} is the space charge electric field resulting from drift-diffusion equilibrium, under the steady-state conditions of illumination [1]. Here we neglect possible thermal ionization (dark conductivity) and saturation of the impurity photorefractive centers in the PRC medium. Those last conditions make equation (2) linear. Notice that $\delta\varepsilon(X,Z)$ is a real quantity, which means that the light absorption is neglected. We define the numeric constant that characterizes the non-linear intensity of diffusion by $\gamma = k_0 n^2 r K_B T e^{-1}$, and the term associated with it in Eq. (2). Here $n=\sqrt{\varepsilon_{\rm a}}$ is the average refractive index of the sample, r is the linear electro-optic coefficient determined by the orientation of the crystal, K_B is the Boltzman constant, T is the absolute temperature, e is the electron charge, and $k_0 = \omega\sqrt{\mu\varepsilon_{\rm a}\varepsilon_0}$ is the light wave number in a optically linear medium.

Using the approximation of slow variation of the amplitude, which is only valid for greater distances than the optical wavelenghts, would result in the following equation:

$$\left|\frac{\partial^2 E(X,Z)}{\partial Z^2}\right| \ll \left|\beta\frac{\partial E(X,Z)}{\partial Z}\right| \qquad (3)$$

and substituting $\delta\varepsilon(X,Z)$ into Eq. (1), we obtain the following differential equation, which in fact proves to be linear.

$$k_0^{-2}\frac{\partial^2 E(X,Z)}{\partial X^2} + D'\frac{\partial E(X,Z)}{\partial X} - i2bk_0^{-1}\frac{\partial E(X,Z)}{\partial Z} - CE(X,Z) = 0 \qquad (4)$$

where $C = (\beta^2 - k_0^2)k_0^{-2}$, $D' = 2\gamma k_0^{-1}$. This equation describes the beam propagation only for $x \geq 0$ region. Assuming that $C = 0$, then $b = -\beta k_0^{-1} = -1$, or $\beta = k_0$.

Using the normalized variables $x = k_0 X$ and $z = 2^{-1}k_0 Z$, then the Eq. (4) results in:

$$\frac{\partial^2 E(x,z)}{\partial x^2} + D\frac{\partial E(x,z)}{\partial x} - ib\frac{\partial E(x,z)}{\partial z} = 0 \tag{5}$$

for $x \geq 0$ region (PRC medium), where $D = 2\gamma$, (in a more general case, for instance when decaying by absorption, b would be more complicated); b is considered real for this case. Now (5) is the normalized equation and $E(x,z)$ is the REAL electric field amplitude, because in this situation this is a problem of the photorefractive eigen-modes.

Besides, Eq. (5) describes propagation only for $x \geq 0$, then for $PRC' - PRC$ media we can re-write it as :

$$\frac{\partial^2 E(x,z)}{\partial x^2} + sign(x)D\frac{\partial E(x,z)}{\partial x} - ib\frac{\partial E(x,z)}{\partial z} = 0 \tag{6}$$

where

$$sign(x) = \begin{matrix} 1 & for\ x \geq 0 \\ -1 & for\ x < 0 \end{matrix} . \tag{7}$$

and for $x < 0$ change D by $-D$.

2.1 Tangential components of electric field

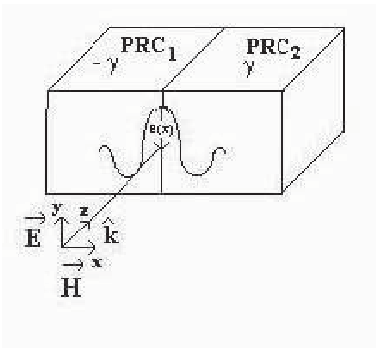

Figure 1: Configuration of the photorefractive media and orientation of the coordinate axis of the problem that is being under consideration.

Figure 1 shows a linear light polarization case, however in this paper we only discuss the $PRC' - PRC$ case. The continuity conditions for this case are:

$$E_1(x,z)|_{x=0} = G_1(z) = E_2(x,z)|_{x=0} = G_2(z) \tag{8}$$

which represents the continuity of electric field components and

$$\frac{\partial E_1(x,z)}{\partial x}|_{x=0} = F_1(z) = \frac{\partial E_2(x,z)}{\partial x}|_{x=0} = F_2(z) \tag{9}$$

represent the continuity of the magnetic field components. Here (8) and (9) are given by (27).

The electric field distribution expressed as $K_1(x)$ for tangential components, is given as:

$$E_1(x,z)|_{z=0} = E_2(x,z)|_{z=0} = K_1(x) \tag{10}$$

Here we limit our consideration only to this case. Even in this case we simplify the analysis by assuming that the boundary between two different media does not posses any particular properties (e.g. surface trapping centers, blocking properties for the charge transfer, surface photogalvanic effects, etc.).

2.2 Differential equation solution.

To solve (5) and/or (6) in order to obtain a non-stationary solution, two methods already known can be applied: the Laplace or the Fourier Transform, however the latter is more complicated to use. For this case we apply the Laplace Transform with respect to the variable z, which results in an in-homogeneous differential equation of second order, for the $x \geq 0$ region we apply the Laplace Transform, that is $\mathcal{L}(E(x,z)) = E(x,s)$ in (5) and/or (6), thus:

$$\frac{d^2E(x,s)}{dx^2} + D\frac{dE(x,s)}{dx} - idsE(x,s) = -ibK_1(x) \tag{11}$$

in (6) for the other region $x < 0$, we replaced $D \rightarrow -D$ in the obtained solution.

Now, s is the new variable; applying the continuity conditions (8) and (9), in (11) we obtain:

$$E(x,s) = \exp(-\frac{Dx}{2})\left[G(s)\cos\left\{\frac{\beta_1(s)x}{2}\right\} + H_1(s)\sin\left\{\frac{\beta_1(s)x}{2}\right\}\right] + g_1(x,s) \tag{12}$$

where

$$H_1(s) = \left[DG(s) + 2F(s)\right]\beta_1^{-1}(s) \tag{13}$$

is the non-stationary solution, and $g_1(x,s)$ is the non-homogeneous solution of (11), in the space (x,s), where:

$$\beta_1 = -\frac{i}{2}\sqrt{D^2 + 4ibs} = -i\sqrt{\gamma^2 + ibs} \tag{14}$$

and

$$g_1(x,s) = 2ib \int_0^x K_1(t) \exp\left[-\frac{D}{2}(x-t)\right] \frac{\sin\left[\beta_1(s)(x-t)\right]}{i\beta_1(s)} dt \tag{15}$$

Now applying the inverse Laplace Transform in (15), i.e.

$$\mathcal{L}_s^{-1} g_1(x,s) = g_1(x,z) \tag{16}$$

in the equation (6), where the solution to the non-homogeneous part is:

$$g_1(x,z) = \sqrt{\frac{ib}{4\pi z}} \exp(-\eta z) \int_0^x K_1(t) \exp\left[-\frac{ib(x-t)^2}{4z} - \frac{D(x-t)}{2}\right] dt \tag{17}$$

where $\eta = D^2(4ib)^{-1}$ produces the phase shift of the beam and

$$\mathbf{G}(x,z,t,0) = \sqrt{b(4i\pi z)^{-1}} \exp\left[-\frac{ib}{2}(t-x)^2(2z)^{-1}\right] \exp\left[\frac{D}{2}(t-x)\right] \tag{18}$$

represents the Kernel function.

Equation (17) is the part that represents the spread integral [12], [13].

$$g_1(x,z) = \exp(-\eta z) \int_0^x K_1(t)\mathbf{G}(x,z,t,0) dt \tag{19}$$

2.3 Spherical Bessel amplitude of the electric field in convolution integrals

From (19) and applying it for a Spherical Bessel field distribution (of the first kind) $J_{\frac{1}{2}}(\mathfrak{a}x)$ the initial condition is being set up. That field distribution was chosen because we want to analyze how the wave propagates along the interface media. This wave is particularly important because it has oscillations, and we would like to know whether they remain the same or if they just simply flat up. According to [1] the author finds out oscillatory surface waves that in real situations do not exist, as it is demonstrated in the solution of the differential equation (5).

For the $PRC' - PRC$ media:

$$K_1(x) = \sqrt{\frac{\pi}{2\mathfrak{a}x}} J_{\frac{1}{2}}(\mathfrak{a}x) = j_0(\mathfrak{a}x) = \frac{\sin(\mathfrak{a}x)}{\mathfrak{a}x} \tag{20}$$

where $\mathfrak{a}$ is the input beam radius, , and where;

$$\mathfrak{a} = \begin{array}{l} \mathfrak{a} \;\; for\ x \geq 0 \\ \text{-}\mathfrak{a} \;\; for\ x < 0 \end{array} . \tag{21}$$

only for conventional use in the solution (27).

Now applying the inverse Laplace Transform on the homogeneous part of (12), its solution is:

$$E(x,z) = \exp(-\frac{Dx}{2})\left[E_I(x,z) + E_{II}(x,z) + E_{III}(x,z)\right] + g_1(x,z) \tag{22}$$

which represents the non-stationary electrical field amplitude, where

$$E_I(x,z) = \frac{\tau x}{2\sqrt{\pi}} \int_0^z \left[G(z-u)\exp(\frac{\tau^2 x^2}{4u} - \eta u)\right] u^{-3/2} du \tag{23}$$

$$E_{II}(x,z) = \frac{D}{2\sqrt{\pi}} \int_0^z \left[G(z-u)\exp(\frac{\tau^2 x^2}{4u} - \eta u)\right] u^{-1/2} du \tag{24}$$

$$E_{III}(x,z) = \frac{1}{\sqrt{\pi}} \int_0^z \left[F(z-u)\exp(\frac{\tau^2 x^2}{4u} - \eta u)\right] u^{-1/2} du \tag{25}$$

The $G(z)$ and $F(z)$ depend on the field distribution $K_1(x)$ and where $\tau = 2i\sqrt{ib}$.

The integrals (23), (24) and (25), represent the convolution of boundary conditions, that we apply to a particular case, the Spherical Bessel distribution.

The integrals (17), (23), (24) and (25), represent the form in which the beam is propagating and determine the spread by diffraction of the beam.

3. Analytical non-stationary solution of integrals for PRC'-PRC solutions.

3.1 Spherical Bessel electrical field amplitude propagation

Evaluating (17), (23), (24) and (25) for the $x \geq 0$ region so that we can get a stable solution, the input beam radius $\mathfrak{a}$ must satisfy;

$$\mathfrak{a} = \frac{D}{2}, \tag{26}$$

i.e. that both parameters are connected (light parameter with diffusion coefficient of the PRC crystal), for a stable solution of the differential equations (5) and/or (6).

Then we obtain the solution of the differential equation (5) for the region $x \geq 0$, that is:

$$E(x,z) = f_0(x,z)\left[f_1(x,z) + f_2(x,z) - f_3(x,z) - f_4(x,z) + f_5(x,z)\right] + f_6(x,z) \tag{27}$$

where

$$f_0(x,z) = \frac{g_0}{\sqrt{\pi z}} \exp(-\frac{Dx}{2} + \frac{ibx^2}{4z}) \tag{28}$$

$$f_1(x,z) = \frac{\sqrt{\pi\tau_1}}{\tau_-} \exp(\frac{\tau_-^2}{4\tau_1}) \left[\text{erf}\left\{ \sqrt{\tau_1}(x - \frac{\tau_-}{2\tau_1}) \right\} - \text{erf}\left\{ \frac{\tau_-}{2\sqrt{\tau_1}} \right\} \right] \tag{29}$$

$$f_2(x,z) = \frac{1}{\tau_-} \sqrt{\frac{\pi\tau_1}{i}} \exp(\frac{i\tau_-^2}{4\tau_1}) \left[\text{erf}\left\{ \sqrt{i\tau_1}(x - \frac{\tau_-}{2\tau_1}) \right\} - \text{erf}\left\{ \frac{\tau_-}{2} \sqrt{\frac{i}{\tau_1}} \right\} \right] \tag{30}$$

$$f_3(x,z) = \frac{\sqrt{\pi\tau_1}}{\tau_+} \exp(\frac{\tau_+^2}{4\tau_1}) \left[\text{erf}\left\{ \sqrt{\tau_1}(x - \frac{\tau_+}{2\tau_1}) \right\} - \text{erf}\left\{ \frac{\tau_+}{2\sqrt{\tau_1}} \right\} \right] \tag{31}$$

$$f_4(x,z) = \frac{1}{\tau_+} \sqrt{\frac{\pi\tau_1}{i}} \exp(\frac{i\tau_+^2}{4\tau_1}) \left[\text{erf}\left\{ \sqrt{i\tau_1}(x - \frac{\tau_+}{2\tau_1}) \right\} - \text{erf}\left\{ \frac{\tau_+}{2} \sqrt{\frac{i}{\tau_1}} \right\} \right] \tag{32}$$

$$f_5(x,z) = (\frac{\tau_1 x}{\tau_+} - \frac{1}{2}) \exp(-i\tau_1 x^2 + i\tau_+ x) - (\frac{\tau_1 x}{\tau_-} - \frac{1}{2}) \exp(-i\tau_1 x^2 + i\tau_- x) \tag{33}$$

$$+\frac{1}{2}(\mathbf{E}_i(\tau_1 x^2 - \tau_- x) - \mathbf{E}_i(\tau_1 x^2 - \tau_+ x)) \tag{34}$$

where for the Exponential function $\mathbf{E}_i$, must be complete for the values of $x > 0$.

$$f_6(x,z) = g_0 \sqrt{\frac{\pi}{z}} \exp(-\frac{Dx}{2} - \frac{ibx^2}{4z}) \left\{ \begin{array}{c} \text{erf}\left[\sqrt{\frac{zi}{b}} \left(\frac{D}{2} + i\mathfrak{a} \right) + \sqrt{\frac{bi}{z}} \frac{x}{2i} \right] \\ -\,\text{erf}\left[\sqrt{\frac{zi}{b}} \left(\frac{D}{2} - i\mathfrak{a} \right) + \sqrt{\frac{bi}{z}} \frac{x}{2i} \right] \end{array} \right\} \tag{35}$$

where;

$$\tau_1 = \frac{b}{4iz}, \;\; \tau_- = (1-i)\mathfrak{a} + \frac{bx}{2iz}, \;\;\; \tau_+ = (-1+i)\mathfrak{a} + \frac{bx}{2iz} \tag{36}$$

and the solution for $x < 0$ change D by $-D$.

This exact solution (27) of Eq. (5) represents the quasi-stable optical field, considering that a small diffraction is spread along the propagation. Thus if the parameter γ is small ($D = 2\gamma k_0^{-1}$), then the beam diffraction can not be compensated as is shown in figure 2.

That leads to a solution which, although to some extent approximate, is also in analytic form and enables the direct interpretation of the obtained results. Numerical iterations of this solution (27), are mainly aimed at the possibility of achieving the formation of the quasi-stable surface wave.

Applying the limit when $z \to \infty$ in the Eq. (27), the quasi stationary solution is obtained, for the spatial surface wave:

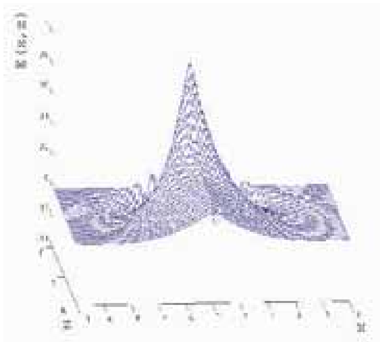

Figure 2: This figure shows the contribution to the diffusion coefficient for $D = 0.5$.The surface-wave represents the transverse profile in the limit $z \to \infty$ as a function of the transverse coordinate x, causing the oscillatory wave tail to extinguish at infinity, and only then followed by the surface wave formation . The beam amplitude in the figure is normalized by 1.

$$\lim_{z \to \infty} E(x, z) = E(x) \tag{37}$$

that is the quasi-steady solution of the soliton-like approximation. See figures 2 and 3.

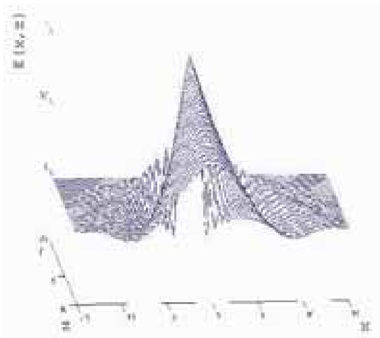

Figure 3: Propagation of a soliton-like (surface wave) in a photorefractive media. In this case the diffusion coefficient is $D = 1$, to reduce the penetration depth of the guided waves. The lobes are smoothened faster when the diffusion nonlinearity of the photorefractive media is increased.

Conclusion

Beam propagation in the non-linear interface is analyzed for the formation of the surface wave. The obtained solution is derived in terms of the dependence of the non-linear intensity of diffusion γ, which has been considered for compensating diffraction effects, as it is shown in the solution of Eq. (27).

Analysis of the results indicate that the nonlinear interface exhibits the guided mode which is formed as an interference pattern of two waves which are self-bent, but only when this self-bending is balanced by the diffusion non-linearity of each medium. In our case with a similar PRC and with the opposite sign of diffusion photorefractive non-linearity ($\gamma' = -\gamma$), these conditions determine us the quasi symmetric surface wave (see figure 2 to 3).

A non-diffracting beam is a Bessel beam with its field amplitude described by a spherical Bessel function of the first kind. The beam has finite extend in the transverse plane and is capable of propagating to infinity in its propagation direction with little beam spreading.

It can be seen that the oscillating surface waves do not exist, as shown in figures 2 and 3, (see e.g. [9], for the case ot the one sample), because the lobes are smoothened when the diffusion nonlinearity of the photorefractive media is increased.

In contrast to the obtained results of the oscillating guided waves found in [1], for this crystal orientation we did not find any evidence that the guided oscillatory waves are produced for both light polarizations, as it is proved in this analysis and in [4, 9], because we used a specific light polarization and an input beam with oscillations that do not remain the same as they propagate along the interface media. Thus ensuring that the oscillating wave tail extinguishes at infinity. The speed in which the oscillations extinguish depends on the diffusion coefficient magnitude as seen in figures **2** and **3**.

Some other important questions arise and are to be investigated in relation to nonlinear photorefractive surface waves, for instance how crystal dark conductivity influences the formation of surface waves (see and compare this study with the results obtained by Stepanov's and Vysloukh's group [3]). They state that the influence of the dark conductivity of the sample is the peculiarity of the formation of a nonlinear transverse eigenmode from an arbitrary input spatial light distribution. Regarding the formation of the surface wave one must take into account the input beam (see Integrals (17), (23), (24) and (25); these integrals are functions of the initial condition which is represented by the input beam. They represent the form in which the beam propagates and they also determine how it spreads out due to the diffraction effect). Moreover the influence of the dark conductivity of the sample is not a consequence of the oscillating wave tail extinguishing at infinity (also see [9]), in contrast to what Stepanov and Vysloukh had stated before.

For a stable wave propagation along the interface of photorefractive media, the nonlinear waves with the parameters $\mathfrak{a}$ and D (where the input beam radius $\mathfrak{a}$ must satisfy $\mathfrak{a} = \frac{D}{2}$, otherwise, when $\mathfrak{a} \neq \frac{D}{2}$, it would be impossible to obtain a stable wave propagation) must be related to each other. The direct interaction between light (represented by the $\mathfrak{a}$ parameter) and matter (represented by the numeric constant that characterizes the non-linear intensity of diffusion $\gamma = k_0 n^2 r K_B T e^{-1}$ of the PRC crystal) is represented by $\mathfrak{a} = \frac{D}{2}$, since that was one of the obtained results that fulfilled the goals of this paper.

Currently the types of transversely nondiffracting beams (surface waves) are under study and shall have an outcome that will illustrate us a bit more of their physical nature and how they could be applied in real life situations.

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