

**THE OSCILLATION MODES IN THE CONFOCAL LASER CAVITY:
THE DIFFRACTION POINT OF VIEW**

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ABSTRACT

The free space electromagnetic field propagation between a spherical emitter and a spherical receiver centered correspond to Fraunhofer diffraction phenomena and is mathematically expressed as a Fourier transform. This approach is used to obtain the beam parameters and show digitally the oscillation modes propagation in the laser cavity.

INTRODUCTION

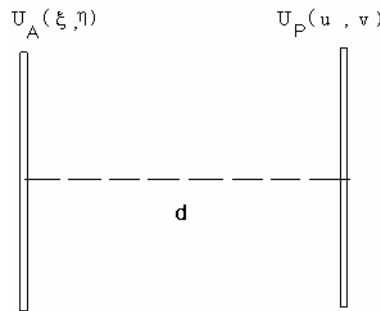


Fig. 1. Free space propagation

In the figure **No 1**, when a planar object is illuminated with a plane wave, the field complex amplitude over the plane detector $U_P(u, v)$ is [1,2] ^[1,2]:

$$U_P(u, v) = \frac{1}{i\lambda d} \exp\left(\frac{-i\pi(u^2 + v^2)}{\lambda d}\right) \iint_{-\infty}^{\infty} \exp\left(\frac{-i\pi(\xi^2 + \eta^2)}{\lambda d}\right) \exp\left(\frac{2i\pi(u\xi + v\eta)}{\lambda d}\right) U_A(\xi, \eta) d\xi d\eta \quad (1)$$

The time origin on P is shifted with respect to the time origin on A, then the factor

$$\exp\left(\frac{-2i\pi d}{\lambda}\right) \quad (2)$$

Has been neglected.

Considering $U_A(\xi, \eta)$ the diffraction plane and $U_P(u, v)$; the emitter or observation plane: We have incorporated the limits of the aperture in the definition of $U_A(\xi, \eta)$.

II. FRESNEL DIFFRACTION AND FOURIER'S SPHERE OPERATOR

The Fresnel diffraction equation can be write of the following form:

$$U_P(u, v) \exp\left(\frac{i\pi(u^2 + v^2)}{\lambda d}\right) = \frac{1}{i\lambda d} \iint_{-\infty}^{\infty} \left[U_A(\xi, \eta) \exp\left(\frac{-i\pi(\xi^2 + \eta^2)}{\lambda d}\right) \right] \exp\left(\frac{2i\pi(u\xi + v\eta)}{\lambda d}\right) d\xi d\eta \quad (3)$$

But the multiplications of the amplitude complex distributions for the phase factors can be interpreted in paraxial approach as complex amplitude distributions over spherical surfaces with radio d for the emitter and radio -d for the receptor.

It is considered that:

$$U_{ASph}(\xi, \eta) = U_A(\xi, \eta) \exp\left(\frac{-i\pi(\xi^2 + \eta^2)}{\lambda d}\right) \quad (4)$$

and

$$U_{PSph}(u, v) = U_P(u, v) \exp\left(\frac{i\pi(u^2 + v^2)}{\lambda d}\right) \quad (5)$$

That is to say, now the complex amplitude distributions are over the spherical surfaces $U_{ASph}(\xi, \eta)$ for the emitter and $U_{PSph}(u, v)$ for the detector; which are tangent to the respective planes. Therefore the Fresnel diffraction can be write as:

$$U_{PSph}(u, v) = \frac{1}{i\lambda d} \iint_{-\infty}^{\infty} U_{ASph}(\xi, \eta) \exp\left(\frac{2i\pi(u\xi + v\eta)}{\lambda d}\right) d\xi d\eta \quad (6)$$

Introducing scaling parameters for the complex amplitude distribution coordinates of the input and the output:

$$\rho = \sqrt{\frac{2\pi}{\lambda d}} \xi, \quad \gamma = \sqrt{\frac{2\pi}{\lambda d}} \eta, \quad \text{and} \quad u' = \sqrt{\frac{2\pi}{\lambda d}} u, \quad v' = \sqrt{\frac{2\pi}{\lambda d}} v$$

That which, can be write as:

$$U_{PSph}(u, v) = \frac{2\pi}{i\lambda d} F[U_{ASph}(\xi, \eta)] \quad (7)$$

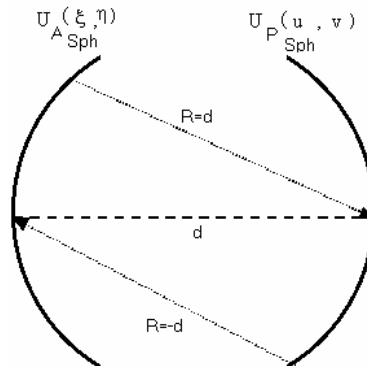


Fig.2. Fresnel diffraction between spherical surfaces input A and output P

Where \mathcal{F} is the Fourier transform; the previous situation is similar to the optic configuration of the figure (2).

The Fresnel diffraction is a Fourier transform between two spherical surfaces whose vertices are located on the same sphere and separate a distance similar to the radius of the sphere. This result from the point of view of the metaxial optics [3] corresponds to the Fourier sphere operator. That is to say $U_{PSph}(u, v)$ is the Fourier sphere of $U_{ASph}(\xi, \eta)$. Obviously that this interpretation fulfills the conservation of the Huygens principle; that is to say that the emitter spherical surface of radius d after the propagation it should be observed at a distance d but on a spherical surface of radius d .

Therefore "If $U_{ASph}(\xi, \eta)$ is a spherical emitter with radii $R=d$, $U_{PSph}(u, v)$ a spherical receiver with radii $R=-d$, and centered over $U_{ASph}(\xi, \eta)$. The field transfer of $U_{ASph}(\xi, \eta)$ to P_{Sph} correspond to Fraunhofer diffraction phenomena and is mathematically expressed as a Fourier transform".

THE CONFOCAL RESONATOR

The most commonly used laser resonators [4,5,6,7] are composed of two spherical mirrors facing each other. The stability of such "open" resonators has been discussed in terms of paraxial rays. To study the modes of laser resonators one has to take account of their wave nature, with mirror apertures that are large compared to the spot size of the beams, a mode of a resonator is defined as a self-consistent field configuration. If a mode can be represented by a wave beam propagating back and forth between the mirrors, the beam parameters must be the same after one complete return trip of the beam.

The configuration of the figure (2). can make correspond with the structure of a confocal resonator, in this case a laser resonator with mirrors of equal curvature is obtained (see Fig. (3)).

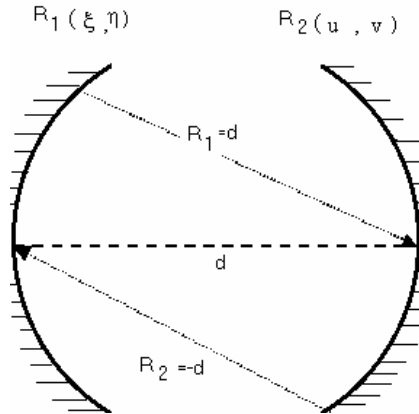


Fig.3. Confocal laser resonator

In the classical theory[6], for this symmetrical structure it is sufficient to postulate self-consistency for one transit of to resonator (which is equivalent to one full period of the lens sequence), instead of a complete return trip. In the proposed theory it should be obtained that the surfaces of the mirrors are coincident with the fronts of phase of resonator modes. Placing a Gaussian function on the surface of the spherical mirror of radius R_1 , with the help of the equation (7). it is found that at the distance d exactly on the mirror of spherical surface R_2 another Gaussian function should be obtained; completing this way, properties of the Fourier transformation. therefore:

$$\exp\left(\frac{-i\pi(u'^2 + v'^2)}{2}\right) = \mathcal{F}\left[\exp\left(\frac{-i\pi}{\lambda d}(\rho^2 + \gamma^2)\right)\right] \quad (8)$$

Inspection of equations (7) and (8) shows that rays passing through a confocal resonator structure are periodically refocused; is equivalent to situation when the input amplitude transmittance is placed in the front focal plane of the lens and the observation plane is equal to the focal length of the lens, in this situation an exact Fourier transform relation between the complex amplitude distribution output and the complex amplitude distribution input is obtained; this means that the stability condition has been reached. In other words the rays passing through a stable resonator are periodically refocused.

Keeping in mind the scaling of coordinates used previously for the spherical mirror R_2 it is obtained that:

$$\exp\left(\frac{-i\pi(u'^2 + v'^2)}{2}\right) = \exp\left(\frac{-i\pi^2}{\lambda d}(u^2 + v^2)\right) \quad (9)$$

For convenience a real beam parameter w is introduced and the following relationship settles down:

$$\frac{\lambda}{\pi \omega^2} = \frac{1}{d} \quad (10)$$

Now it is clear of the equation (9). that w is a real parameter[6] referred to the beam complex parameter, which describes the Gaussian variation beam intensity with the distance to the optic axis; as well as the curvature of the phase front, which has spherical form near the axis. This parameter is denominated the beam radius or "spot size".

One obtains immediately the real beam parameters. One sees that d is equal to the radius of curvature of the mirrors, which means that the mirrors surfaces are coincident with the phase fronts of the resonator modes. The width $2w$ or beam diameter of the fundamental mode is given by

$$\omega^2 = \frac{\lambda d}{\pi} \quad (11)$$

To calculate the beam radius w_0 (beam waist) in the center of the resonator where the phase front is plane, one uses $d/2$ and gets:

$$\omega_0 = \frac{\lambda d}{2\pi} \quad (12)$$

The beam parameters d (confocal parameter) and w describe the modes of all orders. But the phase velocities are different for the different orders, so that the resonant conditions depend on the mode numbers. Resonance occurs when the phase shift from one mirror to the other is a multiple of π .

DIGITAL SIMULATION

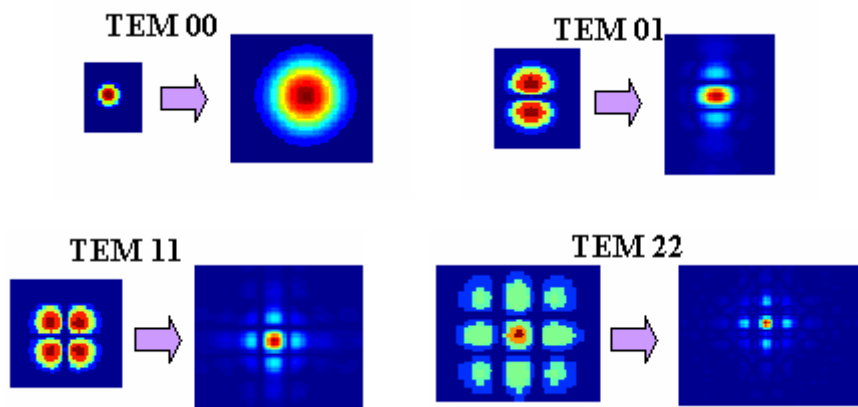


Fig.4. Digital simulation within filtering

For the digital simulation a graphic mathematical algorithm has been used that allows to obtain the Fourier transform; with which is introduced on the spherical surface of the mirror R_1 the figure of a certain order and the calculation allows to obtain the figure diffraction on the spherical surface of the mirror R_2 .

The first results show the images in the introduced oscillation ways and obtained without carrying out any filtrate operation on the image in the resulting way; reason why it is notorious for the case in ways different to the order M_{00} the appearance of other elements that don't correspond to the prospective result, this because the implemented algorithm carries out the Fourier transformation for all the elements introduced in the entrance image as well as its combinations. The fact of the scale change is observed in the coordinates for the image in the exit way with regard to the image in the entrance way; completing this way the property of scaling of the Fourier transformation.

In the case that is carried out filtrate operation in the image in the resulting ways different from the corresponding to M_{00} one can observe the prospective result satisfactorily.

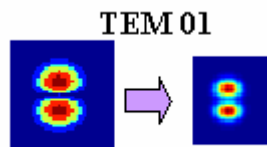


Fig.5. Digital simulation with filtering for TEM_{01}

SUMMARY

It has been proven that the propagation in the oscillation ways in a cavity confocal resonant can be studied starting from the concept of diffraction of Fraunhofer for finite distances of complex widths that are on spherical surfaces. For the case the other cavities our group has proven that the study should be carried out using the concept of fractional Fourier transform.

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